Instructions: Complete each of the following on separate, stapled sheets of paper.

1. Solve each of the following ODEs via variation of parameters (Hint: Complementary solutions?).

(a)
$$y'' + y = \sin(x)$$

(c)
$$y'' + y = \sec^2(x)$$

(e)
$$y'' + y = \sec(\theta)\tan(\theta)$$

(b)
$$y'' + y = \sec(x)$$

(d)
$$y'' + y = \tan(x)$$

(f)
$$y'' + y = \cos^2(x)$$

2. Solve the following ODEs via variation of parameters.

(a)
$$3y'' - 6y' + 6y = e^x \sec(x)$$

(c)
$$y'' + y' - 2y = \ln(x)$$

(b)
$$y'' - 2y' + y = e^x \arctan(x)$$

(d)
$$2y'' + 2y' + y = 4\sqrt{x}$$

3. Solve the following ODEs (Hint: Can you easily reduce the order first?).

(a)
$$y''' + y' = \tan(x)$$

(b)
$$y''' + 4y' = \sec(2x)$$

(c)
$$y''' - 3y'' + 2y' = \frac{e^{3x}}{1 + e^x}$$

4. For each of the following Cauchy-Euler equations, determine all solutions of the form $y = x^m$.

$$(a) xy'' + xy' = 0$$

(c)
$$x^3y''' + xy' - y = 0$$

(b)
$$x^2y'' - 7xy' + 41y = 0$$

(d)
$$x^4y^{(4)} + 6x^3y^{(3)} + 9x^2y'' + 3xy' + y = 0$$

5. Solve the following IVPs.

(a)
$$4y'' - y = xe^{\frac{x}{2}}; \quad y(0) = 1, \ y'(0) = 0$$

(b)
$$2y'' + y' - y = x + 1$$
; $y(0) = 1$, $y'(0) = 0$

6. Study for Midterm 2.