Homework 5

Instructions: Complete each of the following on separate, stapled sheets of paper.

1. Determine whether the following are linearly dependent on $(-\infty, \infty)$.
   (a) $f_1(x) = x, \quad f_2(x) = x^2, \quad f_3(x) = 4x - 3x^2$
   (b) $f_1(x) = x, \quad f_2(x) = e^x, \quad f_3(x) = \cos(x)$
   (c) $f_1(x) = 0, \quad f_2(x) = x, \quad f_3(x) = e^x$
   (d) $f_1(x) = 5, \quad f_2(x) = \sin^2(x), \quad f_3(x) = \cos^2(x)$
   (e) $f_1(x) = \cos(x), \quad f_2(x) = \sin(x), \quad f_3(x) = \sin^2(x)$

2. Verify that the following fundamental sets of solutions to the given ODE; also form the general solution.
   (a) $y'' - y' - 12y = 0;$
   $y_1(x) = e^{-3x}, \quad y_2(x) = e^{4x}$
   (b) $4y'' - 4y' + y = 0;$
   $y_1(x) = \exp(\frac{x}{2}), \quad y_2(x) = x\exp(\frac{x}{2})$
   (c) $x^2y'' - 6xy' + 12y = 0;$
   $y_1(x) = x^3, \quad y_2(x) = x^4$
   (d) $x^2y'' + xy' + y = 0;$
   $y_1(x) = \cos(\ln(x)), \quad y_2(x) = \sin(\ln(x))$
   (e) $x^3y^{(3)} + 6x^2y^{(2)} + 4xy^{(1)} - 4y = 0;$
   $y_1(x) = x, \quad y_2(x) = x^{-2}, \quad y_3(x) = x^{-2}\ln(x)$
   (f) $y^{(4)} + y^{(2)} = 0;$
   $y_1(x) = 1, \quad y_2(x) = x, \quad y_3(x) = \sin(x), \quad y_4(x) = \cos(x)$

3. Verify that the following are general solutions to the given ODEs.
   (a) $y'' - 7y' + 10y = 24e^x;$
   $y = Ae^{2x} + Be^{5x} + 6e^x$
   (b) $y'' + y = \sec(x);$
   $y = A\cos(x) + B\sin(x) + x\sin(x) + \cos(x)\ln(\cos(x))$
   (c) $y'' - 4y' + 4y = 2e^{2x} + 4x - 12;$
   $y = Ae^{2x} + Bxe^2x + x^2e^{2x} + x - 2$
   (d) $2x^2y'' + 5xy' + y = x^2 - x;$
   $y = Ax^{-\frac{1}{2}} + Bx^{-1} + \frac{1}{15}x^2 - \frac{1}{6}x$