Homework 1

Instructions: Complete each of the following on separate, stapled sheets of paper.

1. For each of the below ODEs, state the order of the equation and whether it is linear or nonlinear.
   (a) $(1-x)\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 5y = \cos(x)$
   (b) $x\frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^4 + y = 0$
   (c) $\frac{d^2u}{dr^2} + \frac{du}{dr} + u = \cos(r+u)$
   (d) $\sin(\theta)\frac{d^my}{d\theta^m} - \cos(\theta)\frac{dy}{d\theta} = 2$
2. Verify that the given function is an explicit solution of the given ODE.
   (a) $2dydx + y = 0;\ y = \exp(-\frac{x}{2})$
   (b) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 13y = 0;\ y = e^{3x}\cos(2x)$
   (c) $\frac{d^2y}{dx^2} + y = \tan(x);\ y = -\cos(x)\ln(\sec(x)+\tan(x))$
   (d) $\frac{dy}{dx} + 2xy = 1;\ y = \exp(-x^2)\left(1 + \int_{t=0}^{x} \exp(t^2)\ dt\right)$
3. Verify that the given family of functions is a solution of the given differential equation (all c's are constant).
   (a) $\frac{dP}{dt} = P(1-P);\ P = \frac{ce^t}{1+xe^t}$
   (b) $x^3\frac{d^3y}{dx^3} + 2x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 12x^2;\ y = 4x^2 + c_1x^{-1} + c_2x + c_3x\ln(x)$
4. Verify that the given function is an implicit solution of the given ODE.
   (a) $\frac{dX}{dt} = (X-1)(1-2X); \ln\left(\frac{2X-1}{X-1}\right) = t$
   (b) $2xy\ dx + (x^2-y)\ dy = 0; 2x^2y + y^2 = 1$
5. Verify that the function below is a solution of the ODE $x\frac{dy}{dx} - ny = 0$ on $(-\infty,\infty)$ for all $n > 1$.
$$y(x) = \begin{cases} c_1x^n & \text{if } x \leq 0 \\ c_2x^n & \text{if } x \geq 0 \end{cases}$$
6. Find values of r for which $y = e^{rx}$ is a solution of the following ODEs.
   (a) $5\frac{dy}{dx} = 2y$
   (b) $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$
7. Find values of r for which $y = x^r$ is a solution of the following ODEs.
   (a) $x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 0$
   (b) $x^2\frac{d^2y}{dx^2} - 7x\frac{dy}{dx} + 15y = 0$
8. Determine whether or not the following ODEs have constant solutions $y = c$.
   (a) $\frac{dy}{dx} = y^2 + 2y - 3$
   (b) $(y-1)\frac{dy}{dx} = 1$
   (c) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 6y = 10$
9. Verify that the ODE system $\begin{cases} \frac{dx}{dt} = x + 3y \\ \frac{dy}{dt} = 5x + 3y \end{cases}$ has solution $\begin{cases} x = e^{-2t} + 3e^{6t} \\ y = -e^{-2t} + 5e^{6t} \end{cases}$.