

Instructions: Complete each of the following on separate, stapled sheets of paper.

1. Are the functions $f_1(x) = 5$, $f_2(x) = \sin^2(x)$, and $f_3(x) = \cos^2(x)$ linearly independent?
2. Are the functions $g_1(x) = x$, $g_2(x) = e^x$, and $g_3(x) = \cos(x)$ linearly independent?
3. Find general solutions to each of the following linear ODEs.
 - (a) $16y^{(4)} + 24y^{(2)} + 9y = 0$
 - (b) $y^{(4)} + y^{(3)} + y^{(2)} = -e^{2x} \sin(5x)$
 - (c) $y'' - 2y' + y = x^3 + 4x$
 - (d) $y'' - 2y' + y = e^x \arctan(x)$
 - (e) $y'' + y = \sec^2(x)$
 - (f) $x^3 y''' + xy' - y = 0$
 - (g) $x^2 y'' + xy' - y = \frac{1}{x+1}$
4. Solve each of the following IVPs.
 - (a) $y'' - 4y' - 5y = 0$; $y(1) = 0$, $y'(1) = 2$
 - (b) $y'' + y = 8 \cos(2x) - 4 \sin(x)$; $y(\frac{\pi}{2}) = 1$, $y'(\frac{\pi}{2}) = 0$
 - (c) $4y'' - y = xe^{\frac{x}{2}}$; $y(0) = 1$, $y'(0) = 0$
 - (d) $x^2 y'' - 5xy' + 8y = 8x^6$; $y(\frac{1}{2}) = 0$, $y'(\frac{1}{2}) = 0$
5. Study for Midterm 2.