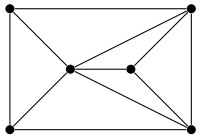
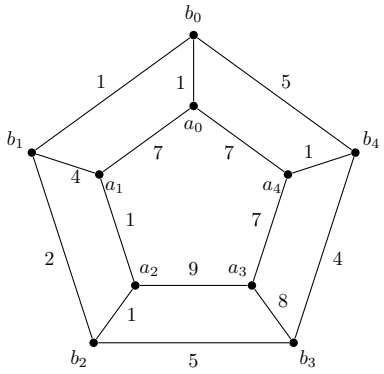
Graph Theory Homework

Instructions: Legibly complete each of the following exercises; +1 bonus point if written in LaTeX.

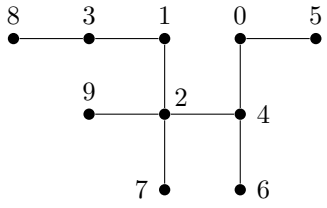
1. Let G be a simple graph with finitely many vertices. Prove that G has an even number of vertices of odd degree.
2. Prove that every graph with at least two vertices has at least two vertices with the same degree.
3. Let G be a simple graph and $v \notin V(G)$. Obtain a new graph $v * G$ from G with vertices $V(v * G) = V(G) \cup \{v\}$ and edges $E(v * G) = E(G) \cup \{\{v, w\} : w \in V(G)\}$. In particular, $v * G$ is the graph obtained by adding a new vertex to G and adding all possible edges from the new vertex to the old vertices.
   (a) Draw $v * C_n$ for $n \in \{3, 4, 5, 6, 7\}$.
   (b) Prove $\chi(v * G) = \chi(G) + 1$ for all graphs G.
4. Compute the chromatic number of the graph below.



5. Prove that the chromatic number of C_n is 2 when n is even and 3 when n is odd.
6. Use Prim's algorithm to compute a minimum weight spanning tree of the weighted graph below.



7. This question concerns the Prüfer code.
   (a) Compute the Prüfer code of the tree given below.



   (b) Compute the labeled tree with Prüfer code $(2, 7, 3, 5, 2, 2, 4, 9)$.