Homework 1

Instructions: Legibly complete each of the following on *stapled sheets of lined* paper.

1. Create a truth table for each of the following propositional statements.

   (a) $(p \to q) \to (q \to p)$

   (b) $(q \leftrightarrow (\neg p)) \vee p$

   (c) $((p \to q) \wedge p) \leftrightarrow (\neg q)$

   (d) $((p \oplus q) \oplus r) \leftrightarrow (p \oplus (q \oplus r))$

2. Use natural deduction to show each of the following argument forms is valid. Use the proof style from class.

   (a) $x \to ((\neg y) \to (\neg z))$, $\neg y$, $y \vee x$ $\therefore \neg z$

   (b) $x \to (x \to (\neg y))$, $y \vee (\neg t)$, x , $w \to t$ $\therefore \neg w$

   (c) $x \to y$, $((\neg x) \vee w) \to k$, $(\neg y) \wedge z$ $\therefore k \vee t$

   (d) $(x \to y) \wedge l$, $(y \to z) \wedge m$, $(x \to z) \to ((x \to y) \to w)$ $\therefore w$

   (e) $(x \wedge y) \vee (z \wedge w)$, $(x \wedge y) \to l$, $(\neg l) \wedge m$, $z \to (n \wedge o)$ $\therefore n$

   (f) $\neg(x \wedge y)$, x , $y \vee z$ $\therefore z$

   (g) $x \wedge (y \vee z)$, $\neg(x \wedge y)$, $(z \wedge x) \to k$ $\therefore k$

   (h) x , $(y \to x) \to z$ $\therefore x \wedge z$

   (i) $x \to z$, $y \to z$ $\therefore (x \vee y) \to z$

   (j) $x \to (y \vee z)$, $\neg y$ $\therefore x \to z$

   (k) $\neg(x \vee (\neg x))$ $\therefore y$

3. Let $A = \{1, 2, 5\}$, $B = \{2, 3, 5, 7\}$. Compute each of the following.

   (a) $A \cup B$ (b) $A \cap B$ (c) $A \setminus B$ (d) $B \setminus A$ (e) $\mathrm{pow}(B)$ (f) $A \times B$

4. Let A, B, and C be sets. Prove each of the following using a string of set equalities.[1]

   (a) $A \cup B = B \cup A$

   (b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

   (c) $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$

   (d) $A \cup B = (A \setminus B) \cup (A \cap B) \cup (B \setminus A)$

   (e) $A \setminus (A \setminus B) = A \cap B$

[1] **Hint**: Write these sets in set-builder notation and manipulate their predicates using the basic logical equivalences.