This document gives you some practice counting, using poker hands as a test case.

A standard poker deck has 52 cards, each with both a rank and a suit. The *suits* are the symbols $C = \clubsuit$, $H = \heartsuit$, $S = \spadesuit$, and $D = \diamondsuit$, and the *ranks* are the symbols A, 2, 3, 4, 5, 6, 7, 8, 9, T = 10, J, Q, and $K.^1$ A *poker hand* is any set of 5 cards from the standard poker deck. There are some special hands in poker, and these have values (i.e. some are better, some are worse). From best to worst, the hands are below.

- (a) A royal flush is a hand T, J, Q, K, A all of the same suit.
- (b) A royal straight is a hand T, J, Q, K, A not all the same suit.
- (c) A straight flush is a hand of five cards in sequence, all in the same suit.
- (d) A four of a kind is a hand with four cards of the same rank.
- (e) A full house is a hand with a three cards of one rank and two cards of a different rank.
- (f) A *flush* is a hand with any five cards in the same suit.
- (g) A straight is a hand with five cards in sequence.
- (h) A three of a kind is a hand with three cards of the same rank.
- (i) A two pair is a hand with two different pairs.
- (j) A pair is a hand with two cards of the same rank.
- (k) A high card hand is a hand which fails all of the above.

The rank of a hand is the highest criterion it satisfies above. Below you will count the ranked hands.

- 1. How many poker hands are there?
- 2. How many royal flushes are there?
- 3. How many royal straights are there?
- 4. How many four of a kinds are there?
- 5. How many three of a kinds are there?
- 6. How many pairs are there?
- 7. How many full houses are there?
- 8. How many two pairs are there?
- 9. How many straight flushes are there?
- 10. How many flushes are there?
- 11. How many straights are there?
- 12. How many high card hands are there?

¹Formally, the deck is the Cartesian product $\{A, 2, 3, 4, 5, 6, 7, 8, 9, T, J, Q, K\} \times \{C, H, S, D\}$.